

On the Effect of the Groove Form on
the Distribution of Light
by a Grating

BY

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The following paper will aim to give a simple treatment of the theory of a plane reflecting grating—that is, a reflecting surface consisting of a great number of equidistant, similar, parallel grooves all lying in one plane—for the case in which each groove is bounded by a finite number of plane faces. Following the presentation of the theory will be given a short discussion of previous work on the subject, comparing such work with the results here obtained in order to bring out the points of agreement or difference. Finally the theory will be illustrated by application to a few numerical problems of practical interest. Experimental work will here be touched on only incidentally, it being the intention of the writers to elaborate this side of the question in a subsequent paper.

GENERAL THEORY

Notation.—We will consider a groove made up of plane portions $AB, BC, \dots$ (Fig. 1) and will generally call such a plane portion a *face* in what follows. We will reckon all directions clockwise from the normal to the grating. We will call

- $c_1, c_2, \dots$ the widths of the faces $AB, BC, \dots$
- $\gamma_1, \gamma_2, \dots$ the directions of the normals to these faces,
- $\pi + \phi$ the direction of the incident light,
- θ the direction of the diffracted light,
- θ_k the direction of the spectrum of the k -th order,
 k being positive when $\theta_k - \theta_0$ is positive,
- a the grating interval.

Simple grating.—By a simple grating we mean a plane grating consisting of alternate strips of transparent and opaque, or of reflecting and non-reflecting, material. The theory of such a grating is given in all works on optics. If the disturbance in any direction θ due to a single slit is of amplitude R_θ and phase δ , we may represent it by a vector in a plane, and the resultant effect of the N slits is obtained by summing these elementary disturbances as vectors. We thus arrive at the result that, if N is large, all the

light is concentrated in the immediate neighborhood of directions θ_k such that $a(\sin \phi + \sin \theta_k) = k\lambda$, where k is the order of the spectrum. The maximum amplitude in these directions occurs when all these elementary disturbances are in phase; it is therefore NR , and hence the maximum intensity is proportional to N^2R^2 . As the factor N is independent of the nature of the slit, it follows that the treatment will apply to any plane grating; and the problem thus becomes one of finding R_θ , the amplitude of the disturbance due to a single groove.

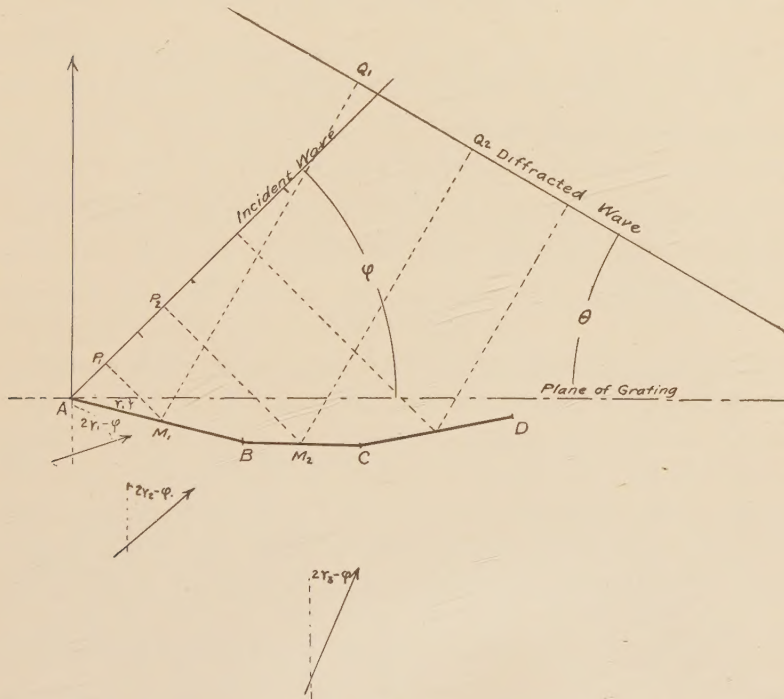


FIG. 1

Only one point needs further mention here. The quantity N^2R^2 is frequently spoken of as being proportional to the "intensity of the spectrum." The term is misleading, and has led to some inaccuracies in the textbooks.¹ The quantity N^2R^2 is the *maximum*

¹ See for example Schuster, *Theory of Optics* (2d ed.), p. 122, where a calculation is made of the efficiency of a grating, neglecting the factor given below.

value of the intensity of illumination in a spectrum line (corresponding to the greatest ordinate in the graph of $y = \sin^2 Nx / \sin^2 x$). But if the resolving power of the grating be sufficient to give us sharp lines, what is actually observed is the *total energy* of the line (corresponding to the area of the above-mentioned curve). To get a number proportional to the total energy we must therefore multiply $N^2 R^2$ by a factor proportional to the width of the line.

Such a factor may be obtained as follows. The difference in path L between the two extreme lines of the grating is

$$L = Na (\sin \phi + \sin \theta).$$

If θ is the direction of a principal maximum we obtain the first minimum on either side by making a small change $d\theta$ such that the corresponding change of path is $\pm \lambda$. We thus have

$$dL = \pm \lambda = Na \cos \theta d\theta,$$

and the width of the line is $2\lambda / Na \cos \theta$; or for a given wavelength the width varies as $\sec \theta$. We thus get for J the "intensity of the spectrum"

$$J = AR_0^2 \sec \theta. \quad (1)$$

It remains to calculate R_0 . If we have a single slit in an opaque screen, the usual formula for the amplitude R is

$$R = R_0 \frac{\sin \alpha}{\alpha}; \quad \alpha = \frac{\pi e}{\lambda} (\sin j + \sin \psi), \quad (2)$$

where j is the angle of incidence, ψ the angle of diffraction, e the slit-width, and R_0 the amplitude in the direction of the incident light. The phase of the disturbance is that at the center of the slit. When $\frac{\pi e}{\lambda}$ is not large, or when we are dealing with directions of diffraction making a considerable angle with the wave normal, the formula is admittedly inadequate, for it takes no account of polarization, and rests on the further assumption that the effect of an element of the wave-front is the same in all directions. The results obtained below will require for their complete accuracy the same limitations; the defect, however, being in no way peculiar to the theory of the composite groove, but belonging equally to the theory of the simple grating as ordinarily given.

General case.—Our method of treating the more general case is a rather obvious extension of the simple theory outlined above. Suppose we have given a groove $ABCD \dots$ consisting of plane faces $AB, BC, CD, \dots$ and that plane waves are incident on it at an angle ϕ . Each face, such as AB , gives rise to a disturbance whose form we may determine as follows: We construct the direction of regular reflection $2\gamma_1 - \phi$ (Fig. 1), and regard AB as a slit in a screen in the plane of AB illuminated by plane waves incident from behind in a direction $2\gamma_1 - \phi$. The amplitude P_θ in any direction θ is obtained from (2) by putting $j = \phi - \gamma_1$, $\psi = \theta - \gamma_1$, namely,

$$P = P_0 \frac{\sin a}{a}; \quad a = \frac{\pi c_1}{\lambda} [\sin(\phi - \gamma_1) + \sin(\theta - \gamma_1)]. \quad (3)$$

The intensity P_0 at the center of the pattern is proportional to the reflecting power of the surface and to the area of the wave-front falling on AB , so that we may write

$$P = A r \cos(\phi - \gamma_1) c_1 \frac{\sin a}{a} \quad (4)$$

where A depends only on the intensity of illumination, and is therefore the same for all the faces $AB, BC, \dots$

The disturbance due to any other face is given by an expression of the same form as (4). Each of these disturbances will have a definite phase and the resultant effect of the whole groove in the direction θ will be found by adding these separate disturbances as vectors. The phase of any one of these component disturbances may be found by taking the sum of the perpendiculars M_1P_1, M_1Q_1 from the center of the groove face to an arbitrary fixed wave-front and an arbitrary fixed plane normal to the direction θ . The

phase is then $\frac{2\pi}{\lambda}$ times this distance, plus the phase-change due to reflection. We thus see that the phase-differences consist of two parts, one depending on the angle of incidence and the other on the angle of diffraction, the two terms being independent of each other. A case of particular interest is offered by the simple triangular groove, with normal illumination on the grating. In this case the difference of path in the direction θ of the disturbances due to the two faces is simply $a/2 \cdot \sin \theta$. But the spectra are diffracted in directions θ such that $a \sin \theta = k\lambda$, so that the phase-relation between

the two parts of the groove changes by half a wave-length as we pass from one order to the next; or if β is the phase-difference between the disturbances in the central image, it will be the same in all the even orders, while in the odd orders it will be $\beta + \pi$.

In case the direction of the incident light is such that it does not reach to the bottom of an angle, we must take as our slit the effective portion of the groove-face. If again the direction of reflection is such that the light meets another face of the groove before emerging, we should consider it as reflected from this second face, and take this face as our aperture. Strictly speaking this treatment is only approximate; we should rather regard the second face as illuminated by the first, considering each element of the first face as a source, so that we would have to deal at the second face with light *diffracted* by the first face, not with light regularly reflected. Since, however, the direction of regular reflection coincides with the diffraction maximum, the above method gives an approximate solution. The difficulty should nevertheless be borne clearly in mind, as we shall see that many of the difficulties of numerical computation find their explanation in the complications introduced by multiple reflection.

In the case of metallic reflection the phase-change due to reflection is a function, not only of the angle of incidence, but also of the plane of polarization. The same is true of the reflecting power of the groove-face. As the diffraction formula used here takes no account of polarization, it would be useless labor in applying the theory to take account of these refinements. In the numerical examples given below we have assumed perfect (or at least uniform) reflecting power, and have neglected the phase-change due to reflection. The practical effect of these simplifications can be seen best in a consideration of the detailed results.

To sum up the preceding: The energy of a spectrum which is diffracted in a direction θ is proportional to

$$R^2 \sec \theta,$$

where R is the amplitude of the disturbance due to a single groove. If the groove consist of a number of plane faces, R will be given

by the vector sum of the disturbances due to these faces. If the width c of a face is such that $\frac{\pi c}{\lambda}$ is not small, and if $\phi - \gamma$ is not large, the amplitude of the disturbance due to this face will be given by

$$P = Ar \cos(\phi - \gamma) c \frac{\sin a}{a} ; \quad a = \frac{\pi c}{\lambda} [\sin(\phi - \gamma) + \sin(\theta - \gamma)] ;$$

and the phase will be a sum of two independent parts, one a function of the angle of incidence, and the other of the angle of diffraction.

PREVIOUS WORK

We have next to consider the relation of the foregoing to previous treatments of the subject. Schuster in his *Theory of Optics* (2d ed., p. 122) considers briefly the question of groove-form in discussing gratings with predominant spectra. He considers a plane face of a groove as giving rise to plane waves of finite extent, and concludes that if these plane waves differ in path by a whole number of wave-lengths they will build up again into a single plane wave, all the light reflected from these faces of the grooves being concentrated into one order. The same point of view is adopted by Trowbridge and Wood¹ in a recent experimental paper, but only as an approximation. Such reasoning ignores the process of diffraction altogether, and is not in our opinion valid. For the only case in which we can consider the groove-face as giving rise to plane waves is when the grating interval is large compared to the wave-length, and in this case the spectra are correspondingly close together, so that the energy is still distributed among several spectra. (The echelon grating furnishes an illustration of this state of things.)

A theory of the effect of groove-form based on the general theory of diffraction is given in a paper by Rowland.² The general method used in this paper is sound, but there are errors in the calculation, so that Rowland's final formula is incorrect, as well as

¹ *Phil. Mag.* (6), **20**, 886, 1910.

² *Astronomy and Astrophysics*, **12**, 129, 1893; *Phil. Mag.* (5), **35**, 397, 1893; *Rowland's Physical Papers*, p. 525.

a number of his conclusions. When corrected, Rowland's method leads to a formula which, when applied to grooves with plane faces, is equivalent to that obtained above, so far as the value of R is concerned. He likewise neglects, however, the distinction between the intensity of illumination and the energy of the spectrum. As the paper is one which has been rather extensively quoted and referred to, and as the errors seem to have hitherto passed unnoticed, it seems worth while to indicate briefly his general method, showing how his results can be made to agree with ours. The general equations will be simplified to conform to the limitations of the present investigation. Taking a grating in the xz plane, with lines parallel to the z axis, and with illumination perpendicular to these lines, we may ignore the z co-ordinate altogether. If x', y' are the co-ordinates of the source A , x, y the co-ordinates of a point P of the grating surface, and x'', y'' the co-ordinates of a point B of the screen, we may represent the disturbance at x'', y'' due to an element dS of the grating surface by an expression of the form

$$Ae^{-\frac{2\pi i}{\lambda}[\overline{AP} + \overline{PB}]}$$

where A is the amplitude, and the imaginary exponent gives the phase. If we are dealing with parallel light, A will not involve the distance. If χ be the angle between dS and AP , and if we ignore variations in the reflecting power of the surface, we may take A equal to $dS \cos \chi$. In Rowland's paper the factor $\cos \chi$ is overlooked, and A is taken as simply equal to dS —an evident error, for the effect of dS must be proportional to the area of the wave-front which it intercepts. With the notation used above, $\tan \phi = \frac{x' - x}{y' - y}$ and $\tan \theta = \frac{x'' - x}{y'' - y}$. Also, if the grating is near the origin as compared with the source and screen, we have approximately

$$\overline{AP} = \sqrt{(x' - x)^2 + (y' - y)^2} = \sqrt{x'^2 + y'^2} - \frac{xx' + yy'}{\sqrt{x'^2 + y'^2}},$$

$$\overline{PB} = \sqrt{(x'' - x)^2 + (y'' - y)^2} = \sqrt{x''^2 + y''^2} - \frac{xx'' + yy''}{\sqrt{x''^2 + y''^2}}.$$

Writing $R = \sqrt{x'^2 + y'^2} + \sqrt{x''^2 + y''^2}$,

$$l = \sin \phi + \sin \theta = \frac{x'}{\sqrt{x'^2 + y'^2}} + \frac{x''}{\sqrt{x''^2 + y''^2}},$$

$$m = \cos \phi + \cos \theta = \frac{y'}{\sqrt{x'^2 + y'^2}} + \frac{y''}{\sqrt{x''^2 + y''^2}},$$

we have $\overline{AP} + \overline{PB} = R - (lx + my)$, an equation which becomes exact for parallel light. We have therefore to integrate

$$\int e^{-\frac{2\pi i}{\lambda}[R - lx - my]} \cos \chi \, ds = B \int e^{\frac{2\pi i}{\lambda}(lx + my)} \cos \chi \, dS$$

over the surface of the grating. As the ruling is periodic this breaks up into a number of equal integrals, one over each groove; so that we have only to consider a single groove. If further the groove be made up of a number of plane faces, we may take one of these faces to be

$$x = -y \cot \gamma + b;$$

whence

$$lx + my = (-l \cot \gamma + m)y + lb =$$

$$\frac{-(\sin \phi + \sin \theta) \cos \gamma + (\cos \phi + \cos \theta) \sin \gamma}{\sin \gamma} y + lb = -\frac{\lambda a}{\pi c \sin \gamma} y + b,$$

where $a = \frac{\pi c}{\lambda} [\sin(\phi - \gamma) + \sin(\theta - \gamma)]$ has the same value as in (3),

$$dx = dy \cot \gamma, \quad ds = dy \csc \gamma, \quad \cos \chi = \cos(\phi - \gamma).$$

The integral thus becomes

$$e^{\frac{2\pi i b l}{\lambda}} \cos(\phi - \gamma) \csc \gamma \int_{u_1}^{u_2} e^{-\frac{2\pi i}{c \sin \gamma} y} dy,$$

where u_1 and u_2 are the ordinates of the edges of the face. The integration gives at once

$$\frac{e^{\frac{2\pi i b l}{\lambda}}}{-2ia} \cos(\phi - \gamma) \left[e^{-\frac{2ia}{c \sin \gamma} u_1} - e^{-\frac{2ia}{c \sin \gamma} u_2} \right].$$

The factor in parentheses can be written

$$-2i \sin \left[\frac{a}{c \sin \gamma} (u_1 - u_2) \right] e^{-\frac{a}{c \sin \gamma} (u_1 + u_2)},$$

but $u_1 - u_2 = c \sin \gamma$, so that we get for the final value of the integral

$$c \cos(\phi - \gamma) \frac{\sin a}{a} e^i \left[-\frac{a(u_1 + u_2)}{c \sin \gamma} + \frac{2\pi b l}{\lambda} \right]. \quad (5)$$

The real part of this expression gives the amplitude, which is that found above (4). To show that the phase is the same, we take a wave-front through the origin, and a plane through the origin perpendicular to the direction of diffraction. The co-ordinates of the center of the face are

$$\bar{x} = b - \frac{u_1 + u_2}{2} \cot \gamma, \quad \bar{y} = \frac{u_1 + u_2}{2},$$

and the distances of this point from the planes just mentioned are

$$\left(b - \frac{u_1 + u_2}{2} \cot \gamma\right) \sin \phi + \frac{u_1 + u_2}{2} \cos \phi,$$

$$\left(b - \frac{u_1 + u_2}{2} \cot \gamma\right) \sin \theta + \frac{u_1 + u_2}{2} \cos \theta.$$

Adding these two distances we have

$$bl + \frac{u_1 + u_2}{2} [-l \cot \gamma + m] = bl - \frac{u_1 + u_2}{2} \frac{\lambda}{\pi c} \frac{a}{\sin \gamma},$$

and multiplying by $\frac{2\pi}{\lambda}$ to reduce to angle, we get $\frac{2\pi bl}{\lambda} - \frac{(u_1 + u_2)a}{c \sin \gamma}$ as found above (5).

NUMERICAL EXAMPLES

We give below a few examples of how the theory works out in simple cases. As a typical groove we will take a simple triangular groove, consisting of two plane faces which make angles of 15° and -30° respectively with the plane of the grating (Fig. 2 a). We will take no account of loss of light by reflection or of phase-change on reflection.

Variation of distribution with wave-length.—In Table I are given the percentages of the total incident energy in the various orders for normal illumination, and for different wave-lengths. The results were obtained by a combination of graphical and numerical methods, the estimated probable error of the computation being about 1 per cent. The wave-lengths are expressed in fractions of the grating interval, the numbers in parentheses below them are the values in $\mu\mu$ which would correspond to a ruling of 10,000 lines to the inch. Three columns are given under each wave-length. The third of these gives the percentage of the total light in the various orders—a quantity which we shall refer to as the

TABLE I
DISTRIBUTION OF ENERGY AS A FUNCTION OF

[illegible]

[illegible]

efficiency of the grating. The first and second columns give the percentages which we should have if the faces BC and AB were blackened successively, all the light being reflected in turn from the 15° and 30° faces respectively. These figures will serve to show in a general way how the phase-relations of the two faces affect the distribution.

A check on the accuracy of the computation is furnished by the condition that the sum of the figures in any column should equal the theoretical total energy. This value is 100 for the columns headed "Total" and 68.3 and 31.7 for the columns headed AB and BC respectively. The agreement is as good as is to be expected from the rather rough methods employed, as far as $\lambda = 0.15a$. Beyond this value there are irregularities, which we will consider briefly. In the first place the values for BC begin to fall short of the theoretical values by considerable amounts. This is due to the fact that the direction of regular reflection from this face of the groove makes such a large angle with the normal to the grating that the center of the diffraction pattern due to this face lies outside the highest order spectrum on that side. The result is that we have light diffracted by this face of the groove in such directions that the energy must be redistributed, by reflection in the other face, among other spectra before it can escape from the grating surface. We are not able to take account of these secondary diffraction phenomena in this elementary treatment, so that this energy is lost to calculation. A second reason for shortage, in the case of the wave-lengths $0.20a$ and $0.25a$, is found in those spectra which escape just at grazing incidence, as this energy must likewise appear in other orders. Finally when the wave-length becomes greater than $0.3a$ the formula (2) becomes quite inaccurate because of the size of the quantity $\frac{\pi c}{\lambda}$.

The effect of the phase-relations of the two faces is greatest, relatively, in the weaker spectra. The phase-relations in a metallic grating would be much more complicated than is assumed here, but the same sort of effect is to be expected in any case, the details of the distribution being of course different.

Another point which is seen immediately is that while the effi-

ciency in the brightest spectrum is a function of the wave-length, it does not increase or decrease continuously with the wave-length, but is periodic. Thus the fifth order for $\lambda = 0.10a$ contains about the same percentage of the total energy as the second order for $\lambda = 0.25a$. The presence of color in the central image is likewise seen to be a property of gratings of this type, it being unnecessary to assume a square groove to explain this phenomenon. It is extremely probable that every form of groove would give such color. The statement made by Rowland (*op. cit.*, p. 6) that in general a simple groove will tend to give a bright first order is not corroborated by this investigation.



FIG. 2

The range of wave-lengths over which a grating will be bright in any one order is a question of some practical interest. Take, for instance, the fourth order in the above table. We find a fairly high efficiency between $0.11a$ and $0.15a$, or between $279 \mu\mu$ and $381 \mu\mu$; this spectrum would be very bright in the ultra-violet. With increasing wave-length the range of high efficiency becomes greater; thus the first order is bright from $762 \mu\mu$ out to the limit of performance of the grating, the exact figures being, however, not obtainable.

Variation of distribution with angle of illumination.—We will now keep a fixed wave-length, and study the effect of varying the angle of incidence. The results of such a calculation are given in Table II, the wave-length chosen being $0.2a$, or green light for a 10,000 ruling.

The figures at the bottom of the table give the theoretical totals for the columns above them. We see that, for all except the first two angles of incidence, the calculated total for the face *BC* falls far short of the theoretical value. The obliqueness of the center of the diffraction pattern is the principal cause of this shortage, to which, however, the small effective width of the face contributes largely. For the angles 30° and 45° the light falling on

TABLE II
DISTRIBUTION OF ENERGY AS A FUNCTION OF THE ANGLE OF INCIDENCE

Order	-30°			-15°			0°			15°			30°			45°			60°		
	AB	BC	Total	AB	BC	Total	AB	BC	Total	AB	BC	Total	AB	BC	Total	AB	BC	Total	AB	BC	Total
-7.....	0.0	0.1	0.1																		
-6.....	0.1	10.4	10.5	0.3	4.9	6.6															
-5.....	0.0	17.8	17.8	0.0	18.8	18.0															
-4.....	0.1	9.3	9.4	0.2	9.2	11.3	0.1	14.9	17.7												
-3.....	0.1	1.4	1.5	0.1	0.8	0.5	0.1	1.7	1.0	0.8	7.7	5.1									
-2.....	0.0	0.1	0.1	0.1	0.2	0.4	0.1	0.1	0.4	0.0	0.0	0.1	1.8	8.7	8.5						
-1.....	0.0	0.7	0.7	0.6	0.6	0.4	0.8	0.5	0.0	0.5	0.4	0.2	0.3	3.4	4.1	2.7	1.2	6.9			
0.....	1.2	0.6	1.8	0.4	0.2	0.9	0.1	0.1	0.4	0.8	0.1	1.3	2.3	0.0	2.2	2.9	1.1	0.6	0.5		0.5
1.....	3.2	0.3	3.5	0.0	0.0	0.0	0.6	0.1	0.3	0.5	0.0	0.4	0.0	0.2	0.2	5.4	0.9	10.1	32.0		32.0
2.....	59.2	0.0	59.2	44.7	0.1	48.1	30.7	0.2	35.5	27.9	0.1	30.5	35.0	0.0	34.7	51.6	0.7	41.9	54.7		54.7
3.....	64.9	40.7	105.4	10.5	0.3	8.4	33.7	0.2	29.3	40.2	0.1	37.9	35.9	0.0	36.3	22.3	0.5	26.0	15.9		15.9
4.....				56.9	35.2	94.6	2.7	0.1	3.9	0.3	0.0	0.4	0.2	0.0	0.2	1.0	0.3	0.7	2.2		2.2
5.....							68.9	17.9	88.5	0.2	0.0	0.1	0.5	0.0	0.5	0.2	0.1	0.6	0.0		0.0
6.....										0.2	0.0	0.2	0.8	0.0	0.8	0.7	0.0	0.5	0.3		0.3
7.....										71.4	8.4	76.2	0.8	0.0	0.9	0.6	0.0	0.6	0.3		0.3
8.....													77.6	12.3	88.4	0.6	0.0	0.3	0.3		0.3
9.....																88.0	5.8	88.2	0.4		0.4
	57.7	42.2		63.4	36.6		68.3	31.7		73.2	26.8		78.8	21.2		86.6	12.0		106.6		106.6
																			100.0		100.0

BC undergoes a second reflection before escaping from the grating. This sends the center of the pattern off somewhat less obliquely, and the figures show a slight improvement. The second of the causes just mentioned has become more important, however.

Two things strike one at once on examining this table. In the first place, we see that over the entire range of angles considered, the orders 2 and 3 are both bright; so that a casual observer looking at the grating would say that the second and third orders were both bright, regardless of the particular angle at which he might happen to hold the grating. On the other hand, the fluctuations of intensity of these two spectra are by no means less noticeable, so that a measurement of the intensities of the spectra obtained by keeping the bolometer or other energy-measuring device fixed, and rotating the grating, would give results which were in no way comparable with those obtained by keeping the grating fixed and moving the bolometer. Measurements of spectral intensities which involve the former procedure do not therefore give true measurements of the *distribution* of energy.¹

It seems worth while to consider briefly the effect of certain variations in the groove-form. If we imagine the groove considered above to have its angle determined by the shape of the ruling point, there are two ways by which with a given point we can alter the character of the ruling. In the first place we may alter the depth of the ruling. In the case considered above the entire original surface of the grating plate has been ruled away; if this is not done there will remain flat portions CD (Fig. 2 *b*) of the original surface between each groove, and we must now regard the groove as including one of these flat portions. The general effect of this change should be clear in the light of what has been said above. The directions of the diffraction maxima of the oblique faces will be the same as before, but these faces being now smaller, the intensities of the disturbances due to them will be cut down. We shall have, besides, a new disturbance with its maximum in the direction of the central image. The general effect will thus be to

¹ See for instance Wood, *Physical Optics*, p. 179; and Trowbridge and Wood, *op. cit.*, p. 6. The inapplicability of the method is evident when we consider that by rotating the grating we get in all twice as many spectra as the grating can give in any one position.

increase the brightness of the central image at the expense of the other spectra. Phase-relations will no doubt complicate this, but not in such a way as to alter the general character of the effect.

A second way of changing the ruling is by tilting the point in the plane of incidence. The general effect of this should likewise be clear—the direction of maximum efficiency will be shifted to other orders, and in addition the orders on one side will be enhanced at the expense of the others owing to the change in the relative area of the two faces (Fig. 2 *c*).

Practical application.—In conclusion we may inquire how far the results obtained above should be applicable to the case of actual gratings. The limitations of the formula for the diffraction of a single slit are obvious; it takes no account of polarization, and is, moreover, strictly applicable only to the case of wide slits. That it gives reasonably consistent results is itself no criterion for its applicability; an examination of the question from the standpoint of electromagnetic theory could alone answer this question. We have at present, however, no better way of treating diffraction phenomena, save in the case of a single diffracting edge, the complete theory of which has been worked out by Sommerfeld. The phase-changes on reflection assumed above are undoubtedly too simple, but correction on this head seems to offer no insuperable difficulties. The foregoing theory is believed to be at least more complete than any heretofore advanced, though undoubtedly capable of much further improvement. The question may be asked finally, how far the assumption of grooves with plane faces is realized in practice. The pictures of a grating groove given in the textbooks—curved and irregular—may perhaps be taken as indications of current ideas on the subject, so that the triangular groove would appear to many too simple. It is certainly true that no matter how irregular the shape of the grating groove, the grating will be good provided the grooves are identical and equally spaced. The experience of one of the writers, however, who has had charge of the ruling engines in this laboratory during the past year, enables us to state with some assurance the conviction that such gratings have seldom if ever been ruled. In the first place the ruling is done, not with a *point*, as is commonly supposed, but with the

edge of the natural diamond; and it is found that unless the edge is so sharp as to possess no visible width under the highest magnifying power the diamond will not rule; further, that a diamond, after continued use, will cease to rule without the edge showing visible bluntness. The metal of the grating is thus probably not gouged or ploughed out, but pressed aside by the edge, and follows in general the shape of this edge. This view is also borne out by the fact that tilting the diamond in the plane of incidence changes the direction of maximum intensity in the manner to be expected on the above theory. If the groove were simply an irregular scratch this were not to be expected.

PHYSICAL LABORATORY
JOHNS HOPKINS UNIVERSITY
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BIOGRAPHICAL NOTE

Carroll Mason Sparrow was born in Baltimore, Maryland, January 10, 1880. He is the son of Leonard Kip Sparrow and Nannie Magill Sparrow. His early training was received in the public schools of Baltimore, which he left in 1896. The years 1897-1899 were spent in commercial life. In the fall of 1899 he was matriculated at the Johns Hopkins University, and studied there until 1902, when he left at the end of the junior year to enter the United States Coast and Geodetic Survey. He resigned from the Government service in 1907 and returned to the Johns Hopkins University, taking the degree of A. B. there in 1908. Since then he has pursued graduate studies in Physics, Mathematics, and Astronomy. He was appointed Fellow for 1909-1910, and Instructor in Physics for 1910-1911. During his residence here as a graduate student he has followed courses in Physics under Professor Ames, Professor Wood, and Dr. Pfund; in Applied Electricity under Professor Whitehead; in Mathematics under Professor Morley and Dr. Cohen; and in Astronomy under Dr. Anderson.

